

Dispersion relation and surface gravity of universal horizons

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Abstract

In Einstein-aether theory, violating Lorentz invariance permits some super-luminal communications, and the universal horizon can trap excitations traveling at arbitrarily high velocities. To better understand the nature of these universal horizons, we use ray tracing method to study their surface gravity in charged Einstein-aether black hole spacetime. Instead of the previous result in Ref. [Phys. Rev. D 89, 064061], our results show that the surface gravity of the universal horizon is dependent on the specific dispersion relation, $\kappa_{UH} = 2(z - 1)\kappa_{uh}/z$, where z denotes the power of the leading term in the superluminal dispersion relation, characterizing different species of particles. And the associated Hawking temperatures also are different with z . These findings, which coincide with those in Ref. [arXiv: 1512.01900] derived by the tunneling method, provide a full understanding of black hole thermodynamics in Lorentz-violating theories.

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I. INTRODUCTION

Invariance under Lorentz transformation is one of the pillars of both Einstein's General Relativity (GR) and the Standard Model of particle physics. However, Lorentz invariance may not be an exact symmetry at all energies [1]. Any effective description must break down at a certain cutoff scale, which signs the emergence of new physical degrees of freedom beyond that scale. For example, the hydrodynamics, Fermi's theory of beta decay [2] and quantization of GR [3] at energies beyond the Planck energy. Lorentz invariance also leads to divergences in quantum field theory which can be cured with a short distance of cutoff that breaks it [4]. This conjecture seems to have been proven by astrophysical observations on high-energy cosmic rays [5]. Einstein-aether theory can be considered as an effective description of Lorentz symmetry breaking in the gravity sector and has been extensively used in order to obtain quantitative constraints on Lorentz-violating

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gravity[6]. In another side, violations of Lorentz symmetry have been used to construct modified-gravity theories that account for Dark-Matter phenomenology without any actual Dark Matter [7].

There are several gravitational theories that violate the Lorentz invariance [1], e.g., Hořava-Lifshitz theory [8], ghost condensations [9], warped brane worlds, Einstein-aether theory [6], etc. In Einstein-aether theory, the background tensor fields break the Lorentz symmetry only down to a rotation subgroup by the existence of a preferred time direction at every point of spacetime, i.e., existing a preferred frame of reference established by aether vector u^a . The introduction of the aether vector allows for some novel effects, e.g., matter fields can travel faster than the speed of light [10], new gravitational wave polarizations can spread at different speeds [11]. And it is the universal horizons that can trap excitations traveling at arbitrarily high velocities. This theory can be also considered as a realization of dynamic self-interaction of complex systems moving with a spacetime dependant macroscopic velocity. As to an accelerated expansion of the universe, this dynamic self-interaction can produce the same cosmological effects as the dark energy [12].

This universal horizon may radiate thermally at a fixed temperature and strengthen a possible thermodynamic interpretation though there is no universal light cone [13] (See also [14] for a different suggestion). Berglund *et al* [15] used tunneling method to study black radiation at the universal horizon in the special choice of $z = 2$, where z denotes the power of the leading term in the nonlinear dispersion relation, characterizing different species of particles. They found that the universal horizon radiates as a blackbody at a fixed temperature though the scalar fields violate local Lorentz invariance. Another different viewpoint is in [14], in which the late time radiation was computed. Cropp *et al* [16] studied ray tracing, found the evidence of Hawking radiation at universal horizon and ray lingering near Killing horizon and, gave the covariant definition of surface gravity of universal horizon. However, it is incorrect that the surface gravity they obtained is independent on the specific dispersion relations, see Appendix for more details.

In Ref. [17], we have used tunneling method to study Hawking radiation of the charged Einstein-aether black hole spacetime. Our results at the Killing horizons confirm the previous ones, i.e., at high frequencies the corresponding radiation remains thermal and the nonlinearity of the dispersion does not alter the Hawking radiation significantly. On the contrary, superluminal particles with $z > 1$ are only created at universal horizons and are radiated out to infinity. Although the radiation is also thermal spectrum, different species of particles in general experience different temperatures.

In this paper, we reconsider ray tracing [16] in Einstein-aether black hole spacetime, and find that, instead of the results of Cropp *et al*, the surface gravity of the universal horizon is indeed dependent on the power z ,

hence for the associated Hawking temperature. These findings may be important for the full understanding Lorentz-violating theories. The rest of the paper is organized as follows. In Sec. II we review the peeling-off surface gravity derived in an usual static black hole spacetime. In Sec. III we extend it to the universal horizon. In Sec. IV, we consider a general superluminal dispersion relation with power z and derive its group velocity. In Sec. V, we give the peeling-off surface gravity of an universal horizon and the relation between it and the covariant surface gravity. In Sec. VI, we present our main conclusions.

II. RAYS PEELING AND SURFACE GRAVITY OF AN USUAL STATIC BLACK HOLE

In this section, we first review the standard description of particle trajectories propagating in an usual static spacetime, with particular attention on their behavior close to the Killing horizon. The most usual textbook notion of surface gravity κ_i is defined by the “inaffinity” of the naturally normalized null geodesics on the horizon of a static spacetime [18],

$$\chi^a \nabla_a \chi^b = \kappa_i \chi^b, \quad (2.1)$$

where χ^a is the Killing vector. The other conception of surface gravity relates to the peeling off properties of null geodesics near the horizon. For stationary Killing horizon these two notions coincide, then in this paper, we pay our attention to this “peeling” surface gravity.

The static black hole metric reads

$$ds^2 = -e(r)dt^2 + \frac{1}{e(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (2.2)$$

While the spacetime is perfectly well behaved there, the coordinates (t, r) becomes singular at horizon $r = r_H$, and they are no longer in a one-to-one correspondence with spacetime events. For the sake of constructing spacetime diagram, one usually changes it into Eddington-Finkelstein coordinates

$$ds^2 = -e(r)dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (2.3)$$

with the transformation (v is the advanced time)

$$dv = dt + \frac{dr}{e(r)}. \quad (2.4)$$

It may also be verified that the (v, r) coordinates cover regions **I** and **II** of the Kruskal diagram.

The ingoing light rays move with $dv = 0$, that is, the coordinate lines can be oriented at 45 degrees. The outgoing rays will be given by

$$\frac{dt}{dr} = \frac{1}{2} \frac{dv}{dr} = \frac{1}{e(r)}. \quad (2.5)$$

If the geodesic is near the metric horizon $e(r_H) = 0$, then we can Taylor expand it as

$$\frac{dr}{dt} = \left. \frac{de(r)}{dr} \right|_{r_H} (r - r_H) + \mathcal{O}(r - r_H)^2. \quad (2.6)$$

Defining a peeling surface gravity

$$\kappa_p = \frac{1}{2} \left. \frac{de(r)}{dr} \right|_{r_H} = \kappa_{KH}, \quad (2.7)$$

where κ_{KH} is the the standard surface gravity at the the Killing horizon in the case of static black hole, then we have

$$\frac{dr}{dt} = 2\kappa_p(r - r_H) + \mathcal{O}(r - r_H)^2. \quad (2.8)$$

For two null geodesics $r_1(t)$ and $r_2(t)$ on the same side of the evolving horizon [19]

$$\frac{d|r_1 - r_2|}{dt} \approx 2\kappa_p |r_1(t) - r_2(t)|. \quad (2.9)$$

This makes manifest the fact that κ_p is related to the exponential peeling off properties of null geodesics near the horizon. It is this quantity κ_p that we have seen is ultimately connected to the temperature of the Hawking flux [20].

III. EXTENSION TO EINSTEIN-AETHER SPACETIME

In this section, we analyze the motion of particles endowed with Lorentz-violating dispersion relations and then we construct the ray trajectories in an aether black hole spacetime. This modified dispersion relations arise due to the interaction of particles with the aether. So the trajectories are not simply the geodesics determined by the given metric.

The static spherically symmetric spacetime of Einstei-aether theory takes the Eddington-Finkelstein coordinates form (2.3) and

$$e(r) = (u \cdot \chi)^2 - (s \cdot \chi)^2. \quad (3.1)$$

The corresponding timelike Killing and aether vectors are

$$\chi^a = (1, 0, 0, 0), \quad u^a = (\alpha, \beta, 0, 0), \quad s^a = (\alpha, \beta + \frac{1}{\alpha}, 0, 0), \quad (3.2)$$

where $\alpha(r)$ and $\beta(r)$ are functions of r only

$$\alpha(r) = \frac{1}{(s \cdot \chi) - (u \cdot \chi)}, \quad \beta(r) = -(s \cdot \chi). \quad (3.3)$$

Then, the metric can be written as $g_{ab} = -u_a u_b + s_a s_b + \hat{g}_{ab}$, where \hat{g}_{ab} is projection tensor onto the spatial two-sphere surface and we have the constraints $u^2 = -1$, $s^2 = 1$, $u \cdot s = 0$.

In this black hole spacetime, the aether time gives the aether one-form

$$ds_u = u_a dx^a = u_v dv + u_r dr = u_v d\tau, \quad (3.4)$$

with

$$\tau = v + \int \frac{u_r}{u_v} dr, \quad (3.5)$$

which slices spacetime into a great number of spacelike slices. There are also some peeling-like behaviors of constant τ slices near the universal horizon. So that a notion of κ_p can be associated to these slices. One can calculate this surface gravity via the theory of Hamiltonian as a dispersion relation in geometrodynamics [16, 21]. The variational principle gives Hamilton's equations for the rates of change

$$\frac{dx^\alpha}{d\lambda} = \frac{\partial \mathcal{H}}{\partial k_\alpha}, \quad \frac{dk_\beta}{d\lambda} = -\frac{\partial \mathcal{H}}{\partial x^\beta}, \quad (3.6)$$

which show that \mathcal{H} must be a constant, independent of the time-like λ along the trajectory of the particle. Its value has to be imposed as an initial condition, $\mathcal{H} = 0$. The four momentum vector must be thrown in the aether frame

$$k_a = -k_u u_a + k_s s_a, \quad (3.7)$$

where $k_u = (u \cdot k) = u^a k_a$, $k_s = (s \cdot k)$ are the aether frame energy and momentum.

Suppose that the dispersion relation is $\omega = f(k_s)$, where $\omega = -k_u$, then the Hamiltonian can be chosen as $\mathcal{H} = [\omega^2 - f^2(k_s)]/2$. Substituting it into above equation (3.6), one can obtain

$$\frac{dt}{dr} = \frac{1}{2} \frac{dv}{dr} = \frac{1}{2} \frac{u^v + v_g s^v}{u^r + v_g s^r}, \quad (3.8)$$

for outgoing particles, where $v_g = \partial\omega/\partial k_s$ is the group velocity of propagating particles. It is just the trajectory for an instantaneously propagating ray. Taylor expanding the trajectory near the universal horizon ($u \cdot \chi = 0$), we obtain

$$\frac{dr}{dt} = 2\kappa_{UH}(r - r_{UH}) + \mathcal{O}(r - r_{UH})^2. \quad (3.9)$$

Then, we can define the surface gravity at the universal horizon via this peeling one,

$$\kappa_{UH} \equiv \frac{1}{2} \frac{d}{dr} \frac{dr}{dt} \Big|_{UH} = \frac{d}{dr} \frac{u^r + v_g s^r}{u^v + v_g s^v} \Big|_{UH}. \quad (3.10)$$

As soon as the modified dispersion relation is given, then one can find the group velocity and lastly obtain the surface gravity of the universal horizon of a given Einstein-aether black hole.

It is interesting that this result can reduce to the usual one (2.7). If $v_g = 1$, the group velocity for the luminal particles, by using Eqs. (3.1-3.3), the above equation can be reduced to

$$\kappa_{KH} = \frac{1}{2} \left. \frac{de(r)}{dr} \right|_{r_{KH}}, \quad (3.11)$$

which is the same as (2.7) at the Killing horizon $e(r_{KH}) = 0$.

IV. DISPERSION RELATION AND GROUP VELOCITY

In this paper, we consider the non-relativistic dispersion relation, given by [22, 23],

$$(-k_u)^2 = k_0^2 \sum_{n=1}^z a_n \left(\frac{k_s}{k_0} \right)^{2n}, \quad (4.1)$$

where a_n 's are dimensionless constants, which will be considered as order of unit in the following discussions [23], and z is an integer. Lorentz symmetry requires $(a_1, z) = (1, 1)$. Therefore, in this paper we shall set $a_1 = 1$. The k_0 is the UV Lorentz-violating scale for the matter [16] or the suppression mass scale [24]. The experimental viable range for the k_0 is rather broad and its value shows the size of Lorentz violation of the given field.

The Killing energy is

$$\Omega = -(k \cdot \chi) = -k_u(-u \cdot \chi) - k_s(s \cdot \chi). \quad (4.2)$$

Near the universal horizon, k_s diverges there and can be parameterized by

$$k_s = \frac{k_0 b(r)}{(-u \cdot \chi)^m}, \quad (4.3)$$

where $b(r)$ is regular and nonzero at the horizon and, m is a constant to be determined. We have

$$(-u \cdot \chi)^{mz-1} \Omega = k_0 \sqrt{a_z} b^z - k_0 b (-u \cdot \chi)^{m(z-1)-1} (s \cdot \chi). \quad (4.4)$$

Hence, m and b can be read off

$$m = \frac{1}{z-1}, \quad \sqrt{a_z} b^{z-1} (r_{UH}) = (s \cdot \chi)|_{UH}. \quad (4.5)$$

From (4.1) and (4.3), the group velocity is

$$v_g = -\frac{\partial k_u}{\partial k_s} \approx z \sqrt{a_z} \left(\frac{k_s}{k_0} \right)_{UH}^{z-1} = \frac{z(s \cdot \chi)|_{UH}}{(-u \cdot \chi)}. \quad (4.6)$$

One can see that the group velocity $v_g \rightarrow \infty$ at the universal horizon $(u \cdot \chi)(r_{UH}) = 0$, and is dependent on the specific form of dispersion relation, i.e., parameter z .

The trajectory

$$\begin{aligned} \frac{dv}{dr} &= \frac{u^v + v_g s^v}{u^r + v_g s^r} = \frac{1}{(s \cdot \chi) - (u \cdot \chi)} \cdot \frac{1 + v_g}{-(s \cdot \chi) - (u \cdot \chi)v_g} \\ &\approx \frac{z}{z-1} \frac{1}{(s \cdot \chi)|_{UH}(-u \cdot \chi)}, \end{aligned} \quad (4.7)$$

near universal horizon $(u \cdot \chi) = 0$. The surface gravity at the universal horizon is

$$\kappa_{UH} = \frac{1}{2} \frac{d}{dr} \frac{dr}{dt} \Big|_{UH} = \frac{z-1}{z} (s \cdot \chi) \frac{d(-u \cdot \chi)}{dr} \Big|_{UH}, \quad (4.8)$$

which depends on the specific dispersion relation.

V. SURFACE GRAVITY OF A CHARGED EINSTEIN-AETHER BLACK HOLE

In this section we can calculate the surface gravity at the universal horizon of a charged Einstein-aether black hole [25]. The static spherically symmetric spacetime of Einstein-Maxwell-aether theory takes the Eddington-Finkelstein coordinates form (2.3). When coupling constants $c_{14} = 0$ ($c_{14} \equiv c_1 + c_4$, respectively),

$$\begin{aligned} (s \cdot \chi) &= \frac{1}{\sqrt{g}} \frac{r_{UH}^2}{r^2} \sqrt{\frac{1}{3} \left(1 - \frac{Q^2}{r_{UH}^2}\right)}, \quad r_0 = \frac{2}{3} r_{UH} \left(2 + \frac{Q^2}{r_{UH}^2}\right), \\ (u \cdot \chi) &= -(1 - \frac{r_{UH}}{r}) \sqrt{1 + \frac{r_{UH}}{3r} \left(1 - \frac{Q^2}{r_{UH}^2}\right) \left(2 + \frac{r_{UH}}{r}\right)}, \end{aligned} \quad (5.1)$$

where $g \equiv 1 - c_{13}$, r_0 is a parameter related to the total mass of the aether black holes, and r_{UH} is the location of the universal horizons. When coupling constants $c_{123} = 0$,

$$\begin{aligned} (u \cdot \chi) &= -1 + \frac{r_{UH}}{r}, \quad r_0 = 2r_{UH}, \\ (s \cdot \chi) &= \frac{r_{UH}}{r} \sqrt{\frac{p}{g} - \frac{Q^2}{gr_{UH}^2}}, \end{aligned} \quad (5.2)$$

where $p \equiv 1 - c_{14}/2$. We apply the aether coupling constants condition $0 < c_{14} < 2$, $2 + c_{13} + 3c_2 > 0$, $0 \leq c_{13} < 1$.

Putting everything together in (4.8), we finally obtain the surface gravity for the case of $c_{14} = 0$,

$$\kappa_{UH} = \frac{z-1}{z\sqrt{3gr_{UH}}} \sqrt{\left(1 - \frac{Q^2}{r_{UH}^2}\right) \left(2 - \frac{Q^2}{r_{UH}^2}\right)}. \quad (5.3)$$

If $z = 2$ and $Q = 0$, they become the result in Ref. [13], $T_{UH} = \kappa_{UH}/2\pi = \sqrt{2}/[4\pi r_{UH}\sqrt{3g}]$, which were obtained in Painlevé-Gullstrand (PG) coordinates. In the case of $c_{123} = 0$, the surface gravity is

$$\kappa_{UH} = \frac{z-1}{zr_{UH}} \sqrt{\frac{p}{g} - \frac{Q^2}{gr_{UH}^2}}. \quad (5.4)$$

If $z = 2$ and $Q = 0$, it becomes the result in Ref. [13], $T_{UH} = \kappa_{UH}/2\pi = \sqrt{\mathfrak{p}}/[4\pi r_{UH}\sqrt{g}]$.

Now we consider the comparison of this peeling-off surface gravity to a covariant definition one, which is the normal derivative to the horizon of the redshift factor. For the Killing horizon, the redshift factor is χ^2 . However for the universal horizon it is $(u \cdot \chi)$ which is constant on the universal horizon and captures the role of the aether in the propagation of the physical rays [16]. This definition is

$$\kappa_{uh} = \frac{1}{2}u^a \nabla_a (u \cdot \chi) \Big|_{UH} = \frac{1}{2}(s \cdot \chi) \frac{d(-u \cdot \chi)}{dr} \Big|_{UH}. \quad (5.5)$$

The relation between both definitions is

$$\kappa_{UH} = \frac{2(z-1)}{z} \kappa_{uh} \quad (5.6)$$

One can see that the peeling-off surface gravity κ_{UH} isn't equal to the normal surface gravity κ_{uh} in general. But it is exactly equal to the associated temperature $T_{UH} = \kappa_{UH}/2\pi$ derived in Ref. [17] by tunneling method.

VI. SUMMARY

In Einstein-aether theory, violating Lorentz invariance permits some super-luminal communications, and the universal horizon can trap excitations traveling at arbitrarily high velocities. The trajectories of these super-luminal rays are no longer metric geodesics due to the presence of the aether. In this paper we consider rays' peeling behaviors in the charged Einstein aether black hole spacetime. We first review the peeling-off surface gravity derived by Cropp *et al* [16] and, have found there is a mathematical error (in Appendix). We then modify this process in Einstein-aether spacetime and find that it can reduce to the usual result (2.7) for luminal particles, i.e., their group velocity $v_g = 1$.

By using the superluminal dispersion relation, we use this improved peeling-off surface gravity to the charged Einstein-aether black hole. Instead of the previous result in Ref. [16], our results show that the surface gravity of the universal horizon is indeed dependent on the specific dispersion relation, $\kappa_{UH} = 2(z-1)\kappa_{uh}/z$, where κ_{uh} is the covariant surface gravity (5.5), z denotes the power of the leading term in the nonlinear dispersion relation, characterizing different species of particles. And the associated Hawking temperatures also are different with z . These findings, which coincide with those in Ref. [17] derived by the tunneling method, provide a full understanding of black hole thermodynamics in Lorentz-violating theories.

Appendix A: Review of the incorrect trajectory

In Ref. [16], the authors calculate the value of surface gravity of the universal horizon which is the relevant one for rays propagating with infinite group velocity v_g . However, there is a mathematical error in their deriving which ultimately leads to an incorrect result. We firstly review their original process.

Generically any particle propagating in the Einstein aether spacetime will have a four-velocity that can be given in the orthogonal frame provided by u^a and s^a as

$$V^a = u^a + v_g s^a. \quad (\text{A1})$$

The trajectory for an instantaneously propagating ray would then be given by

$$\frac{dv}{dr} = \frac{V^v}{V^r} = \lim_{v_g \rightarrow \infty} \frac{u^v + v_g s^v}{u^r + v_g s^r} = \frac{s^v}{s^r}. \quad (\text{A2})$$

The group velocity v_g is dependent on the specific form of dispersion relation $\omega = f(k_s)$, and $v_g = \partial\omega/\partial k_s = f'(k_s)$. So one can see that above trajectory is indeed independent on the specific superluminal dispersion.

Then the surface gravity of the universal horizon is

$$\kappa_{uh} = \frac{1}{2} \frac{d}{dr} \left(\frac{s^v}{s^r} \right) \Big|_{uh}, \quad (\text{A3})$$

which is also independent on the specific superluminal dispersion relation.

But, there has a mathematical error in (A2). In the numerator, both u^v and s^v are finite and nonzero at the universal horizon, so the u^v can be ignored. However, in the denominator, s^r is zero at the universal horizon, therefore the product of $v_g s^r$ maybe finite and u^r *cannot be ignored*! In their paper, u^r, v_g , and s^r are given by

$$u^r = -(s \cdot \chi), \quad s^r = (-u \cdot \chi), \quad v_g = 1 + 3l^2 k_s^2 \sim 1 + 3 \frac{(s \cdot \chi)|_{uh}}{(u \cdot \chi)}. \quad (\text{A4})$$

Hence, the product is

$$v_g s^r = (-u \cdot \chi) - 3(s \cdot \chi)|_{uh}, \quad (\text{A5})$$

which is indeed finite at the universal horizon $(u \cdot \chi) = 0$ and, u^r *cannot be ignored*. Ultimately the result (A3) is incorrect.

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